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Q-ball collisions in the MSSM: gauge-mediated supersymmetry breaking

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Abstract

Collisions of non-topological solitons, Q-balls, are considered in the Minimal Supersymmetric Standard Model where supersymmetry has been broken at a low energy scale via a gauge mediated mechanism. Q-ball collisions are studied numerically on a two dimensional lattice for a range of Q-ball charges. Total cross-sections, as well as fusion and geometrical cross-sections are calculated. The total and geometrical cross-sections appear to converge with increasing charge. The fusion cross-section has been estimated to be larger than 60% of the geometrical cross-section for large balls.

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1 Introduction

Various field theories can support stable non-topological solitons [1], Q-balls [2]. A Q-ball is a coherent scalar condensate that carries a conserved charge, typically a $U(1)$ charge. Due to charge conservation, the Q-ball configuration is the ground state in the sector of fixed charge. Q-balls may have physical importance because the supersymmetric extensions of the Standard Model have scalar potentials that are suitable for Q-balls to exist in the theory. In particular, lepton or baryon number carrying Q-balls are present in the Minimal Supersymmetric Standard Model (MSSM) due to the existence of flat directions in the scalar sector of the theory [3, 4].

If supersymmetry (SUSY) is broken at low energy scales by a gauge mediated mechanism, the scalar potential is completely flat for large enough field values. Therefore the energy per unit charge decreases like $B^{-1/4}$ (B is the baryon number) and for large enough B one can have completely stable B-balls since there are no light enough baryon number carrying particles the Q-ball can decay into [5]. If SUSY breaking is due to a hidden supergravity sector in the theory, the potential is not flat. Radiative corrections allow for Q-balls to exists but they are unstable and decay typically to quarks and nucleons [4, 6]. The decay progresses by evaporation from the surface of the Q-ball [8, 9].

Q-balls can be cosmologically significant in various ways. Stable (or long living) Q-balls are natural candidates for dark matter [5]. Their decay offers a way to understand the baryon to dark matter ratio [6] and the baryon asymmetry of the universe [4]. Q-balls can also protect the baryons from electroweak sphalerons [4] and may be an important factor in considering the stability of neutron stars [7].

For Q-balls to be cosmologically significant one needs to have a mechanism that creates them in early stages of the evolution of the universe. Q-balls can be created in the early universe from an Affleck-Dine (AD) condensate [4, 5, 6]. This process has been studied recently by numerical simulations [10, 11] where both the gauge- and gravity-mediated SUSY breaking scenarios were considered. In both cases Q-balls with various charges were seen to form from the condensate.

Collisions of Q-balls have been considered previously in various potentials [11]-[17]. In [11] collisions were simulated on a one dimensional lattice in the gravity-mediated case and it was found that Q-balls typically merge, exchange charge or pass through each other [11]. To our knowledge, collisions have not been studied in the gauge-mediated case previously and the gravity-mediated case has been studied in more than one dimension only in [17]. In a recent paper Q-ball collisions were studied numerically in one, two and three dimensions in a polynomial potential [16]. The main qualitative features of the collision processes were similar in all the three cases.

Since the charge of Q-ball can change in a collision process, they may play an

important role in the evolution of the Q-ball charge distribution after their formation. On the other hand, the Q-ball charge distribution affects the cosmological role that Q-balls may have in the evolution of the universe. The effect of collisions can therefore be an important factor in evaluating the cosmological role of Q-balls.

In this paper we have studied Q-ball collisions in the gauge-mediated scenario on a two dimensional lattice. The gravity-mediated scenario has been analyzed in a previous paper [17].

2 Q-ball solutions

Consider a field theory with a U(1) symmetric scalar potential, $U(\phi)$, with a global minimum at $\phi = 0$. The complex scalar field ϕ carries a unit quantum number with respect to the $U(1)$-symmetry. The charge and energy of a field configuration ϕ in D dimensions are [1]

$$Q = \frac{1}{i}\int(\phi^*\partial_t\phi - \phi\partial_t\phi^*)d^Dx \tag{1}$$

and

$$E = \int[|\dot{\phi}|^2 + |\nabla\phi|^2 + U(\phi^*\phi)]d^Dx. \tag{2}$$

The single Q-ball solution is the minimum energy configuration in the sector of fixed charge. The Q-ball will be stable against radiative decays into ϕ-scalars if condition

$$E < mQ, \tag{3}$$

where m is the mass of the ϕ-scalar, holds. It is then energetically favourable to store charge in a Q-ball rather than in form of free scalars.

Finding the minimum energy is straightforward using Lagrange multipliers. The Q-ball can be shown to be of the form [1]

$$\phi(x,t) = e^{i\omega t}\phi(r), \tag{4}$$

where $\phi(x)$ is now time independent and real, ω is the Q-ball frequency, $|\omega| \in [0, m]$ and ϕ is spherically symmetric.

The charge of a Q-ball with spherical symmetry in D-dimensions is given by

$$Q = 2\omega\int\phi(r)^2d^Dr \tag{5}$$

and the equation of motion at a fixed ω is

$$\frac{d^2\phi}{dr^2} + \frac{D-1}{r}\frac{d\phi}{dr} = \phi\frac{\partial U(\phi^2)}{\partial\phi^2} - \omega^2\phi. \tag{6}$$

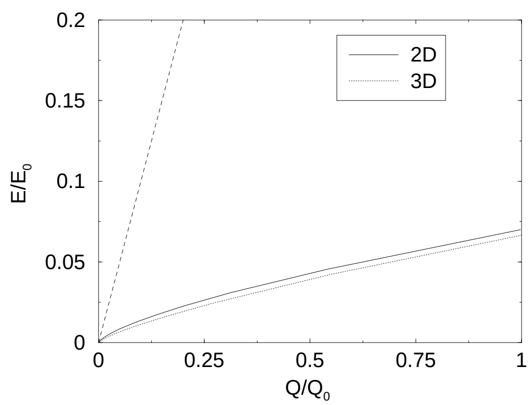
To find the Q-ball solution we must solve (6) with the boundary conditions $\phi'(0) = 0,\ \phi(\infty) = 0$.

In the present paper we consider a potential of the form [5]

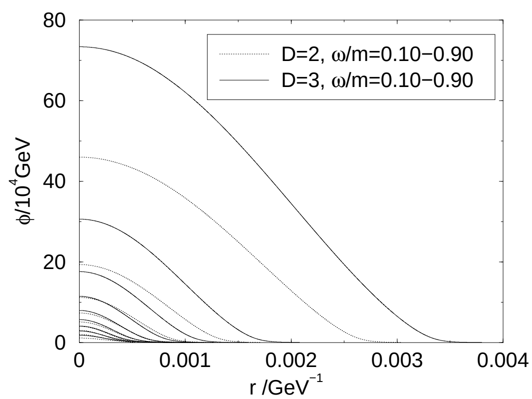
$$U(\phi) = m^4(1 + \log(\frac{\phi^2}{m^2})) + \frac{\lambda^2}{M^2}\phi^6, \tag{7}$$

with parameter values $m = 10^4$ GeV, $\lambda = 0.5$ and $M = 2.4 \times 10^{18}$ GeV. This corresponds to a potential along a flat direction that has been lifted by soft supersymmetry-breaking terms. Supersymmetry is broken here by a gauge-mediated mechanism as opposed to the gravity-mediated case analyzed previously [17].

We have calculated the charge and energy of Q-balls for different values of ω. Energy vs. charge curves are shown in Figure 1(a). The axis scales are chosen differently for two and three dimensions; for two dimensions, $Q_0 = 240(m/\text{ GeV})^2$, $E_0 = mQ_0$ GeV and for three dimensions, $Q_0 = 4.0(m/\text{ GeV})^2$, $E_0 = mQ_0$ GeV. The dashed line is the stability line, $E = mQ$, that indicates that the Q-balls considered here are stable with respect to scalar decays.



(a)



(b)

Figure 1: Q-ball energy as a function of charge and Q-ball profiles in two and three dimensions.

As from Fig. 1(a) can be seen the two and three dimensional energy vs. charge curves follow each other very closely. Also the Q-ball profiles in two and three dimensions have very similar shapes, Fig. 1(b). The similarities in the energy vs. charge curves and Q-ball profiles gives an indication that collisions studied in two and three dimensions are likely to give similar results. In [16] it was also found that the two and three dimensional cases possess similar qualitative features. In contrast, the one dimensional case is fundamentally different from the higher dimensional cases. As from (6) can be seen, there is no dissipation term in one spatial dimension. Processes can then have properties that are not seen in higher dimensions. In the one dimensional simulations we have studied this also seemed to be the case.

Even though the general features of collision processes are similar in different potentials, the exact form of the potential is still important. From the basis of our simulations it appears that the details of collisions are dependent on the choice of potential.

One should also point out that in three dimensions new, ring-like intermediate states were seen in collisions at high velocities [16]. The corresponding process in two dimensions is a right angle scattering -process that we did not observe in our simulations. However, at high velocities, $\mathcal{O}(0.1)$, there can be new types of processes that we have not seen in our low velocity simulations.

3 Collisions

We have studied collisions of Q-balls with equal charges in the potential (7). The range of ω for which numerical simulations have been done is $\omega/m = 0.15,\ 0.20,\ 0.30,\ 0.60$. In terms of charges these values of ω in the two dimensional case correspond to $5.6, 2.1, 0.47, 0.032$ (in units of $(\frac{m}{\mathrm{GeV}})^2$). The initial velocities of the balls are allowed to have two values, $v = 10^{-3}$ or $v = 10^{-2}$.

Collisions are studied over different relative phases, $0 \leq \Delta\phi \leq \pi$. Here we have defined the relative phase, $\Delta\phi$, to be the difference in individual Q-ball phases at the point where the distance between them would be at a minimum assuming there were no interaction between them. If the balls are of equal size, the point at which the relative phase difference is defined is irrelevant. In general, however, one needs to define the phase difference so that it is independent of the initial positions of the balls.

The position of a Q-ball is defined by the location of its maximum amplitude. We have varied the impact parameter to study the scattering cross-sections. As from Fig. 1 can be seen, the Q-balls studied here have thick walls so that one needs a definition for their size. In the gravity mediated case [17] we defined the size of a ball by a Gaussian fit. Here, however, we have found that the Gaussian is a much poorer fit than in the gravity-mediated case. Instead we use a kink solution, $\phi = A + B\tanh(Cr + D)$, and find that it approximates the numerical profiles well. The radius of the ball is defined as $R = \frac{1}{C}(\tanh^{-1}(-\frac{A}{B}) - D)$. This definition has the advantage that as the profile of the ball approaches the thin-wall solution, radius becomes defined in a natural way. It is worth noting that even though the profiles visually appear to be similar to the Q-ball profiles in the gravity-mediated scenario, they are fundamentally different from them and approach a purely thin-walled profile in the large charge limit.

A two dimensional, typically a $\sim 300 \times 300$, lattice with continuous boundary conditions was used in all calculations.

3.1 Numerical Results

As in [17], the collision processes that we have observed can be roughly divided into three categories; fusion, charge exchange and elastic scattering. Fusion is defined as a process where most of the initial charge is in a single Q-ball after the collision and the rest of the charge is lost either as radiation or as small Q-balls. By charge exchange we mean a process where Q-balls exchange some of their charge while the total amount of charge carried by the two balls is essentially conserved. An elastic scattering is defined to be a scattering process where less than 1% of the total charge is exchanged.

The type of a collision process is mainly dependent on the relative phase difference, $\Delta\phi$, between the colliding Q-balls. When $\Delta\phi$ is small the Q-balls fuse and form a larger ball. Excess charge is lost in form of radiation and small lumps of charge. As $\Delta\phi$ increases the Q-balls no longer fuse and start to scatter while exchanging a significant amount of their charge. The amount of charge that is exchanged in a collision decreases with increasing phase difference until the Q-balls scatter elastically. The change from a fusion process to an elastic scattering is typically rapid so that a significant amount of charge is exchanged only at a very narrow range of $\Delta\phi$. This is a different behavior from the gravity-mediated case where charge exchange was generally a much more dominant process [17]. The qualitative effects of changing the relative phase difference are the same for the whole range of ω:s and initial velocities that we have studied.

The effect of the impact parameter is much less pronounced than the phase difference on the type of collision. Typically if the balls fuse at zero impact parameter, they continue to fuse with an increasing impact parameter until at some point the balls start to scatter elastically or while exchanging little of their charge. The interaction probability, averaged over the different phases, also has a clear cut-off with respect to the impact parameter.

From the simulations we can now calculate the total, fusion and geometric cross-sections averaged over the relative phases. The charge exchange cross-section is typically much smaller than the other cross-sections and is not quoted here. The total cross-section includes all the different types of processes. The quoted cross-sections are the three dimensional cross-sections with the interaction radius taken from the two dimensional simulations.

The geometric, total and fusion cross-sections for $v = 10^{-3}$ are plotted in Fig. 2 and in Fig. 3 for $v = 10^{-2}$. In each case we have fitted a curve $\sigma = A(\omega/m)^B$ through the points. From the figures it is clear that the cross-sections increase with decreasing ω. This is naturally due to the increasing size of the Q-balls with decreasing ω. More importantly, one can also see that the ratio of the total cross-section to the geometrical cross-section decreases with decreasing ω *i.e.* increasing charge. This reflects the fact that as the profile of the Q-ball approaches the thin-wall type, the geometrical cross-section is an increasingly better approximation to the total cross-section. This is

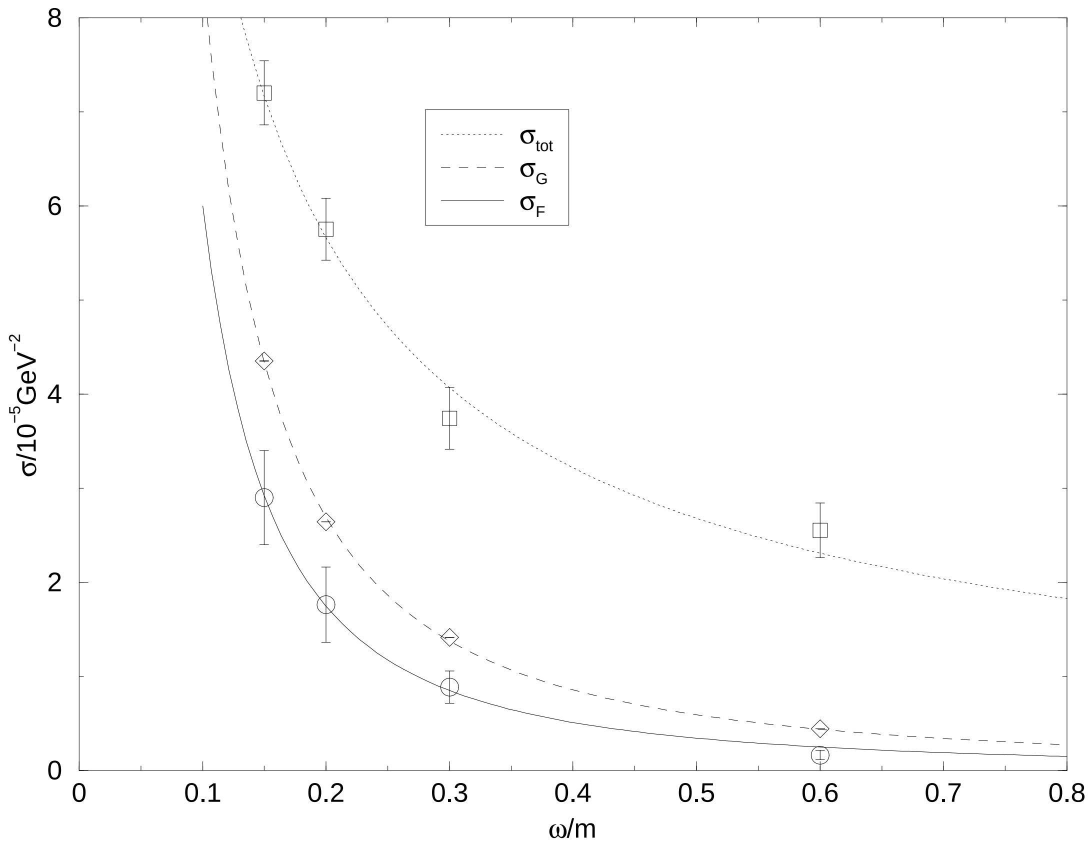


Figure 2: The geometric, total and fusion cross-sections for different values of ω, $v = 10^{-3}$.

intuitively clear; as the balls become more and more thin-walled, the effects of the boundary diminish and the geometrical cross-section dominates the total cross-section. The ratio of the fusion cross-section to the geometrical cross-section is increasing, but only very slightly, with increasing charge. Extrapolating to large, thin-walled balls one can therefore conclude that the total cross-section of large Q-balls is well approximated by their geometrical cross-sections. The fusion cross-section of such balls can then be bounded by a fraction of the geometrical cross-section, $\sigma_F/\sigma_G \gtrsim 0.6$. The geometrical cross-sections of large Q-balls can be estimated analytically, $R \approx \frac{1}{\sqrt{2}} m^{-1} Q^{1/4}$ [18] so that $\sigma_G \approx 2\pi m^{-2} Q^{1/2}$.

The effect of the increased velocity can also be seen from the figures. As the initial velocity is increased tenfold to 10^{-2}, the total and fusion cross-sections both decrease. This is expected since faster balls have less time to interact with each other and are more likely to pass without interacting.

Compared to the gravity-mediated case [17], the results presented here show both some similarities and some differences. In the gravity-mediated case the radius of the balls, and hence the geometrical cross-section, was approximately constant whereas here the radius of the ball varies greatly which obviously has an effect on the different cross-sections. Also the probabilities of different types of processes are different in

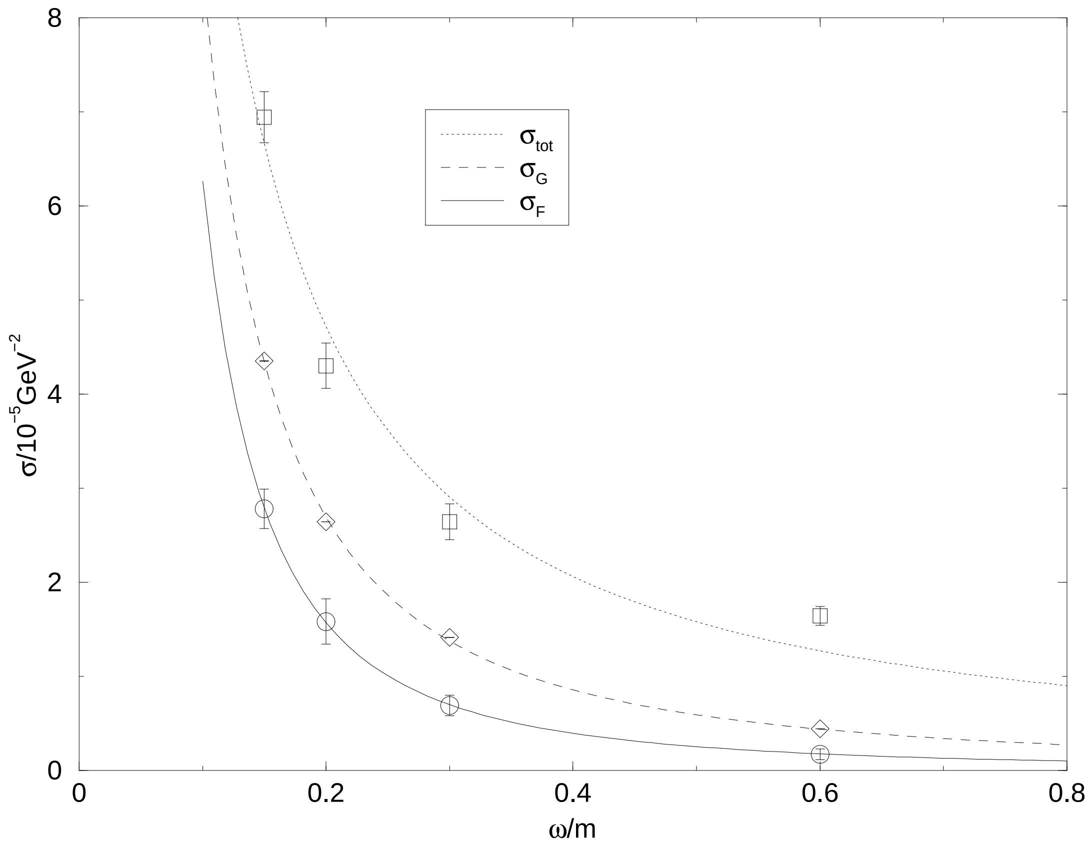


Figure 3: The geometric, total and fusion cross-sections for different values of ω, $v = 10^{-2}$.

the two scenarios: in the gravity-mediated case a charge exchange-process is much more likely to occur than a fusion process, here the charge exchange cross-section is much smaller. The fusion processes also can be distinguished in the two cases, in the gravity-mediated case charge was typically lost as small Q-balls whereas here most of the charge is lost as radiation and lumps of charge. This demonstrates clearly that the exact form of the potential is significant even though the general qualitative features are alike. One can also spot similarities between the gravity- and gauge-mediated scenarios; in both cases the fusion cross-sections increase with increasing balls and an increase in velocity decreases the total cross-sections.

4 Conclusions

In this paper we have studied Q-ball collisions in the MSSM with SUSY broken by a gauge mediated mechanism. It was found that Q-balls may fuse, exchange charge or scatter elastically in a collision depending on the relative phase difference between them. The probability of each process is dependent not only on the relative phase difference but also on the size of the balls. Larger balls are more probable to fuse in

a collision whereas smaller balls are more likely to scatter either elastically or while exchanging some of their charge. Collisions can therefore alter the charge distribution of Q-balls quite significantly, provided that collisions are frequent enough.

Our simulations give an indication that the total cross-section, $\sigma_{\rm tot}$, approaches the geometrical cross-section, σ_G, as the Q-ball size grows so that in the thin-wall limit the total and geometrical cross-sections are equal. The fusion cross-section also grows with the ball size and on the basis of our results we can give an estimate for the fusion cross-section of large balls, $\sigma_F \gtrsim 0.6\ \sigma_G$.

If collisions are to play a significant role in cosmology, the interaction rate must be large enough at some time in the evolution of the universe. If the average interaction rate is smaller than the Hubble rate after the Q-balls are formed, the distribution will 'freeze out' and collisions will not alter the charge distribution. If, on the other hand, the collision rate is initially larger than the Hubble rate, collisions can affect the charge distribution until the distribution freezes out due to the expansion of the universe. Furthermore, if fusion processes dominate, the number density of Q-balls may decrease rapidly which can also freeze the Q-ball charge distribution.

On the basis of our results, the size and the relative phase differences are important in determining the evolution of the Q-ball charge distribution in the gauge mediated scenario. If the Q-balls are initially in the same phase, they typically fuse in a collision. The average size of a Q-ball then increases while the number density decreases. As our results show, the fusion probability increase with increasing Q-ball size so that larger balls are more likely to increase their size even further by collisions. The Q-ball interaction rate is dependent on several factors; clearly the initial Q-ball charge distribution, the interaction cross-section, number density and the velocity distribution affects the average interaction rate.

Q-balls may play a significant role in the past and present stages of the universe. To be able to address the question of their significance more decisively one needs to consider not only the initial charge distribution but also its evolution. Collisions may be an important factor and to study their effect in more detail poses an interesting question that motivates further study.

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